mittent in visibility, that which was opposite to the left hand of the spectator being the most so.

After 11 o'clock the appearance became more and more obscure, until the cross was replaced by an obscure blur. The paraselenæ also disappeared, while the halo, though diminished in brightness, continued some time longer.

June 9, 1845.

CAPTAIN LARCOM, R. E., Vice-President, in the Chair.

James Claridge, Esq., Adolphus Cooke, Esq., Windham Goold, Esq., Charles Croker King, M. D., and Charles Wye Williams, Esq., were elected Members of the Academy.

Mr. Richard Sharpe read a notice of a new electric clock, on the principle of Mr. Wheatstone's telegraphic instruments.

The Rev. Charles Graves made a further communication relative to Algebraic Triplets, and their Geometric Interpretation.

Besides the system of algebraic triplets developed in former communications to the Academy, Mr. Graves has conceived another, which appears to admit of an interpretation in some respects more closely analogous to Mr. Warren's geometrical representation of imaginary quantities.

As the symbol $\sqrt{-1}$ may be taken to indicate a rotation in the plane of x from the axis of x to the axis of y, it seems natural to conceive another symbol representing a rotation in the plane of xz, from the axis of x to the axis of z. The repetition of either of these operations would reverse the direction of a right line originally placed on the axis of x: and

therefore they are both equally fitted to serve as geometric representations of the square root of negative unity. But what is more, there is an infinite number of geometric operations of which this is equally true. For instance, rotation through two right angles in any plane passing through the axis of x would reverse the direction of a line placed upon that axis.

Let us take then two symbols, i and j, denoting *distinct* distributive operations, such that

$$i^2(1) = j^2(1) = -1 : ij(1) = ji(1),$$

and form with them and the three real magnitudes x, y, z the expression

$$x + iy + jz + ij\frac{yz}{x}.$$

As it depends upon three quantities, it may be looked upon as a *triplet;* whilst it is, in some sense, a *quadruplet*, being made up of units of four different kinds : for there is reason to regard $ij(1)$ as an imaginary unit, differing both from $i(1)$ and $j(1)$.

Before we proceed to consider the results arrived at in the multiplication of such triplets, it will be convenient to change their form. For this purpose let us put

$$x = m\cos\phi\cos'\chi, \quad y = m\sin\phi\cos\chi, \quad z = m\cos\phi\sin\chi;$$

the expression $x + iy + jz + ij\frac{yz}{x}$ will thus be transformed into $m(\cos\phi\cos\chi + i\sin\phi\cos\chi + j\cos\phi\sin\chi + ij\sin\phi\sin\chi)$, which is evidently equivalent to $me^{i\phi + j\chi}$.

If then we call m the modulus, and ϕ and χ the amplitudes of the triplet, it will appear that *the modulus of the product of two triplets will be equal to the product of the moduli of the factors : and each amplitude of the product will be equal to the sum of the corresponding amplitudes in the factors.*

The modulus and amplitudes of the triplet (x, y, z) are derived from its constituents by the equations

$$m^2 = x^2 + y^2 + z^2 + \frac{y^2 z^2}{x^2}$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right) \; : \; \chi = \tan^{-1}\left(\frac{z}{x}\right).$$

Hence if x, y, z be the rectangular coordinates of a point in space, the modulus of the right line drawn to it from the origin is a fourth proportional to the projections of this line upon the axis of x and the planes of xy and xz. And the amplitudes are the angles between the axis of x and these two last projections.

The construction thus obtained for the product of two right lines obviously coincides with Mr. Warren's in the case where $z = 0$.

The nullity of the triplet $x + iy + jz + ij\,\frac{yz}{x}$ involves the three equations, $x = 0$, $y = 0$, $z = 0$.

But, however well these triplets fulfil the requisitions of multiplication, we find they will not stand the test of addition. The sum of two such triplets is not necessarily a triplet; nor can we add two of them together, unless they happen to have a common amplitude.

What has been here said may readily be extended, for we might develope the expression $e^{i\phi + j\chi + k\psi + \text{\&c.}}$, in which $i, j, k,$ &c. represent $(n - 1)$ distinct square roots of negative unity, into a series of terms, such as

$$\begin{aligned} & \mathrm{A} + i\mathrm{B} + j\mathrm{C} + k\mathrm{D} + \ldots \\ & + jk\,\frac{\mathrm{CD}}{\mathrm{A}} + ki\,\frac{\mathrm{BD}}{\mathrm{A}} + ij\,\frac{\mathrm{BC}}{\mathrm{A}} + \ldots \\ & + ijk\,\frac{\mathrm{BCD}}{\mathrm{A}^2} + \ldots \end{aligned}$$

and, conversely, we may reduce a multiplet

$$\begin{aligned} & a + ib + jc + kd + \ldots \\ & + jk\,\frac{cd}{a} + ki\,\frac{bd}{a} + ij\,\frac{bc}{a} + \ldots \\ & + ijk\,\frac{bcd}{a^2} + \ldots \end{aligned}$$

*

depending upon n constituents a, b, c, d, &c., to the form

$$me^{i\phi+j\chi+k\psi+\ldots},$$

The modulus m, and the amplitudes ϕ, χ, ψ, ... of the multiplet, will be found by the equations

$$m^2 = a^2 + b^2 + c^2 + d^2 + \frac{c^2d^2}{a^2} + \frac{b^2d^2}{a^2} + \frac{b^2c^2}{a^2} + \frac{b^2c^2d^2}{a^4} + \ldots$$

$$\phi = \tan^{-1}\left(\frac{b}{a}\right), \quad \chi = \tan^{-1}\left(\frac{c}{a}\right), \quad \psi = \tan^{-1}\left(\frac{d}{a}\right) \ldots\ldots\ldots$$

The nullity of the multiplet involves the n equations

$$a = 0, \quad b = 0, \quad c = 0, \quad d = 0, \text{ \&c.}$$

What has been already proved in the case of distinct square roots of negative unity, may be applied, *mutatis mutandis*, to multiplets of the form $me^{i\phi+j\chi+k\psi+\cdots}$, in which i, j, k, &c. are used to denote wholly distinct geometric, or purely imaginary n^{th} roots of positive or negative unity.

Mr. Graves noticed that triple integrals such as $\iiint V\, dxdydz$ may sometimes be advantageously transformed, by putting $x = m\cos\phi\cos\chi$, $y = m\sin\phi\cos\chi$, and $z = m\cos\phi\sin\chi$: the element $dxdydz$ will then be replaced by $m^2\cos\phi\cos\chi\, dmd\phi\, d\chi$.

On the other hand, if we put

$$x = m\,\mathrm{cotr}[\phi, \chi], \quad y = m\,\mathrm{tres}[\phi, \chi], \text{ and } z = m\,\mathrm{tres}[\chi, \phi],$$

we should transform the same integral into $\iiint Vm^2dm\,d\phi\,d\chi$.

Mr. Petrie gave an account of an inscription on an ancient Irish tombstone at Athlone.

Mr. Mallet read extracts from letters by the Rev. Dr. Robinson and others, relating to suggestions for the improvements in working atmospheric railways.

A letter was read from Messrs. Hodges and Smith, stating that, contrary to their directions, 500 copies of Mr. Petrie's volume on the Round Towers had been printed, instead of